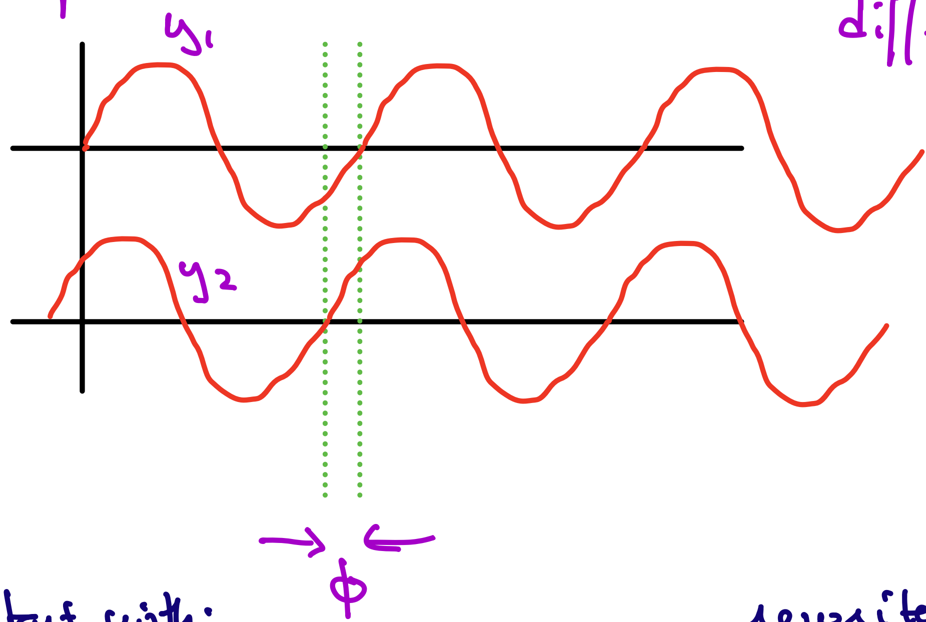
Interference always depends on phase difference of the waves

Recap: waves in 1-Dimension with phase difference ϕ



Start with: rewrite as:

$$y_1(x,t) = A\sin(kx-\omega t) \Rightarrow A\sin(kx-\omega t+\tfrac{1}{2}\phi-\tfrac{1}{2}\phi)$$

$$y_2(x,t) = A\sin(kx-\omega t+\phi) \Rightarrow A\sin(kx-\omega t\ \tfrac{1}{2}\phi+\tfrac{1}{2}\phi)$$

use $\sin(a\pm b)=\sin(a)\cos(b)\pm\cos(a)\sin(b)$

let $a = kx-\omega t+\tfrac{1}{2}\phi$, $b=\tfrac{1}{2}\phi$

$$y_{tot} = y_1+y_2 = \overset{y_1}{A\sin(a-b)} + \overset{y_2}{A\sin(a+b)}$$

$$= A\left[\sin(a)\cos(b)-\cos(a)\sin(b)\right. \quad y_1$$

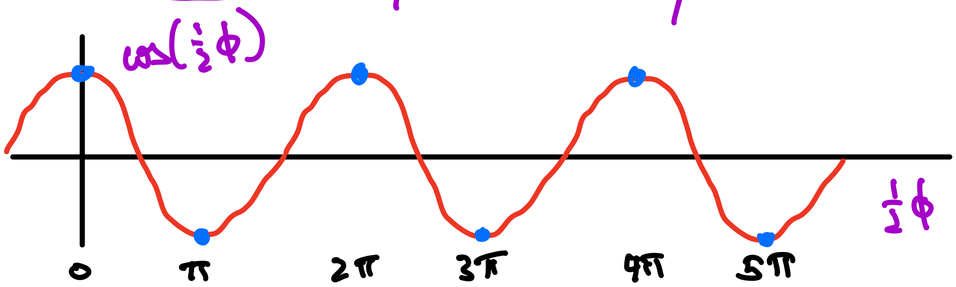
$$\left.+\sin(a)\cos(b)+\cos(a)\sin(b)\right] \quad y_2$$

$$= 2A\cos(b)\sin(a)$$

so $y_{tot} = \underbrace{2A\cos(\tfrac{1}{2}\phi)}_{\text{amplitude}} \underbrace{\sin(kx-\omega t+\tfrac{1}{2}\phi)}_{\text{new wave}}$

y_{tot} amplitude depends on phase difference ϕ

<u>constructive interference</u>: amplitude is max



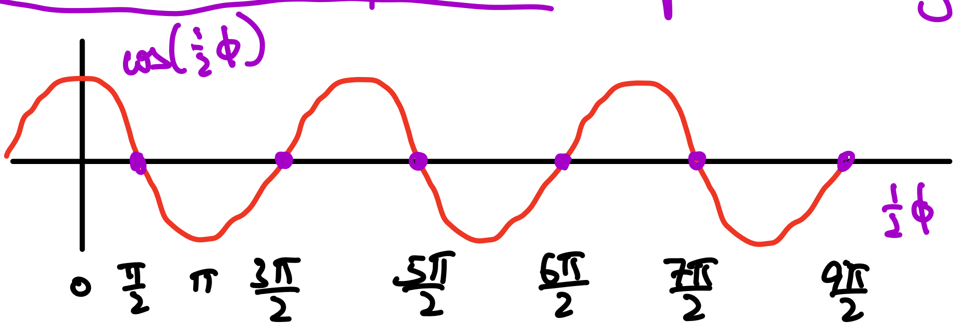
$\cos(\tfrac{1}{2}\phi) = \pm 1 \Rightarrow \tfrac{1}{2}\phi = 0, \pi, 2\pi, \ldots$

$\phi = 0, 2\pi, 4\pi, \ldots$

$= n \cdot 2\pi \qquad n = 0, 1, 2, \ldots$

so if phase difference between 2 waves is a multiple of 2π, y_{tot} has maximum amplitude

<u>destructive interference</u>: amplitude is zero



$\cos(\tfrac{1}{2}\phi) = 0 \Rightarrow \tfrac{1}{2}\phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$\phi = \pi, 3\pi, 5\pi \ldots$

$= n\pi \quad n = 1,3,5, \ldots$ odd

let $n = 2m+1$

then $n=1, m=0$

$=3, m=1$

$\vdots$

write $\phi = (2m+1)\pi \quad , m = 0,1\ldots$

$= (m+\frac{1}{2}) 2\pi \quad , m=0,1\ldots$

$= n\cdot 2\pi + \frac{1}{2} \quad n=0,1\ldots$

phase for:

constructive interference:

$\phi = 2\pi n \qquad n = 0,1,2\ldots$

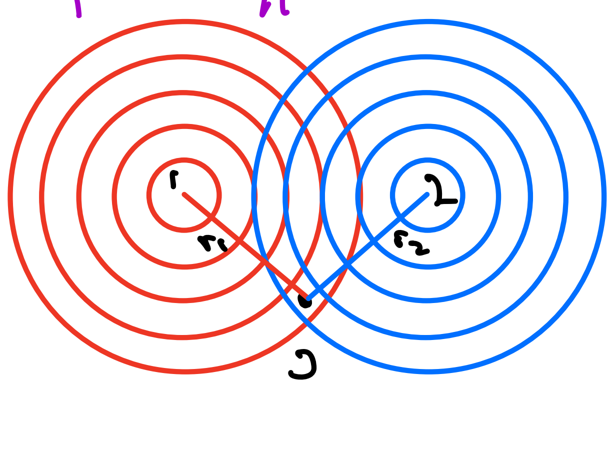
destructive interference:

$\phi = 2\pi n + \frac{1}{2} \quad n=0,1,2\ldots$

Review of phase differences

lots of ways to have phase differences:

1. constant phase ϕ
2. path difference $\Delta r = r_2 - r_1$



$\phi = k\Delta r$

phase diff $=$ $k \times \Delta r$

3. frequency diff: ω_1 & ω_2, $\omega_1 \neq \omega_2$

$$\left.\begin{aligned} y_1 &= A\sin(k_1 x - \omega_1 t) \\ y_2 &= A\sin(k_2 x - \omega_2 t) \end{aligned}\right\} v_1 = v_2 = \frac{\omega}{k}$$

so $k_1 = \frac{\omega_1}{v}$ & $k_2 = \frac{\omega_2}{v}$

$$y_1 = A\sin\left(\frac{\omega_1}{v}x - \omega_1 t\right)$$

$$= A\sin\left[\omega_1\left(\frac{x - vt}{v}\right)\right]$$

x, v, t are the same for both waves

so let $t' = \frac{x - vt}{v}$

then $y_1 = A\sin(\omega_1 t')$

$y_2 = A\sin(\omega_2 t')$

then define $\Delta\omega = \omega_2 - \omega_1$

$2\bar{\omega} = \omega_2 + \omega_1$

solve for ω_1, ω_2: $\omega_1 = \bar{\omega} - \frac{1}{2}\Delta\omega$

$\omega_2 = \bar{\omega} + \frac{1}{2}\Delta\omega$

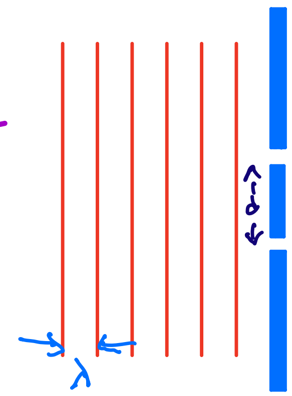
then $y_1 = A\sin\left[\left(\bar{\omega} - \frac{1}{2}\Delta\omega\right)t\right]$

$y_2 = A\sin\left[\left(\bar{\omega} + \frac{1}{2}\Delta\omega\right)t\right]$

$$y_{tot} = y_1 + y_2 = \underbrace{2A\cos\left(\tfrac{1}{2}\Delta\omega t\right)}_{\text{"beats"}}\sin(\bar{\omega}t)$$

2-slit interference

incoming light (plane waves) →



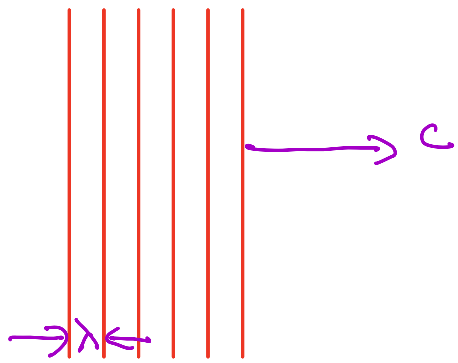
hits screen with 2 slits separated by distance d

slit width is comparable to wavelength

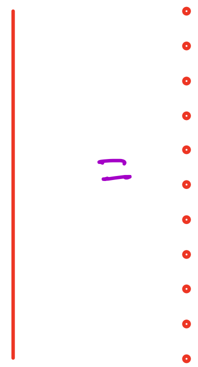
what happens on other side (right side) of slit?

⇒ Huygens Principle:

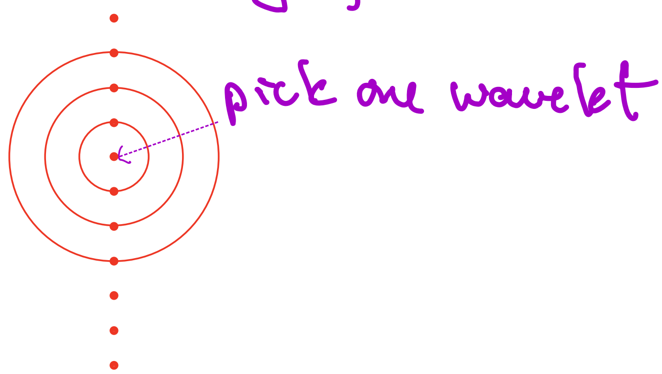
start with light wave front on a plane wave



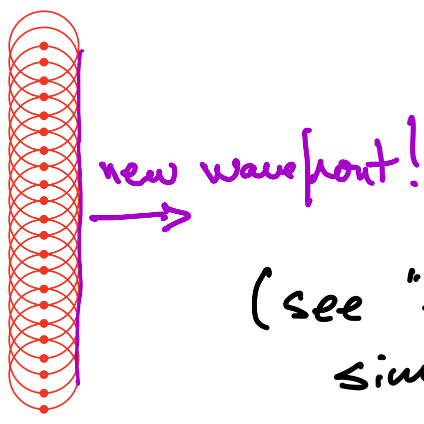
Huygens theory: decompose wavefront into "wavelets"



Then let each wavelet send out spherical waves with the same wavelength, in all directions



now add wavelets from all sources



(see "Huygens Principle" simulation)

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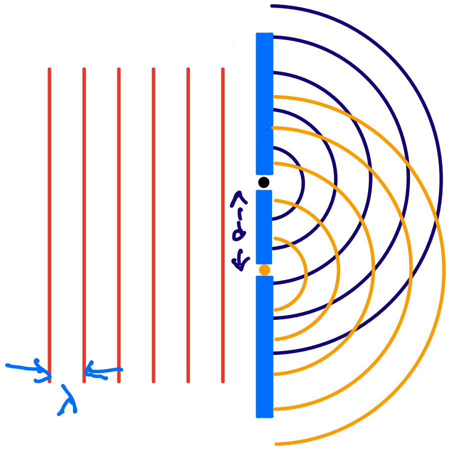
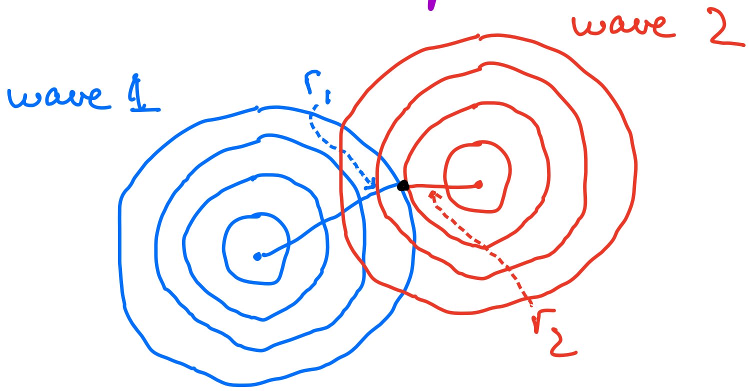
2-slit interference

incoming light (plane waves) →



wavelet at each slit is all that gets through
⇒ slits act like source of 2 waves

⇒ 2 waves interfere at any point on right due to path difference



Condition for CONstructive interference:
⇒ phase diff $k\Delta r = 2\pi n$

Condition for DEstructive interference:
⇒ phase diff $k\Delta r = 2\pi(n + \frac{1}{2})$

using $k = \frac{2\pi}{\lambda}$ we have conditions based on path difference:

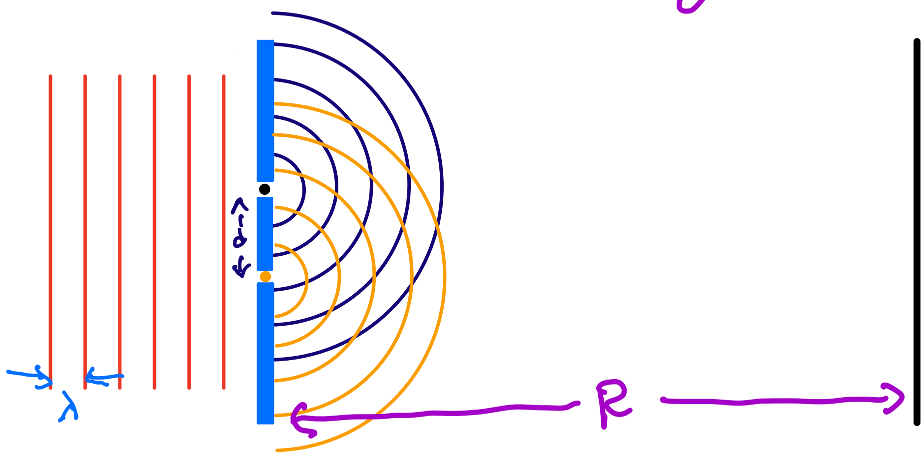
CONstructive: $\frac{2\pi}{\lambda}\Delta r = 2\pi n$

$\boxed{\Delta r = n\lambda}$

DEstructive: $\frac{2\pi}{\lambda}\Delta r = 2\pi\left(n+\frac{1}{2}\right)$

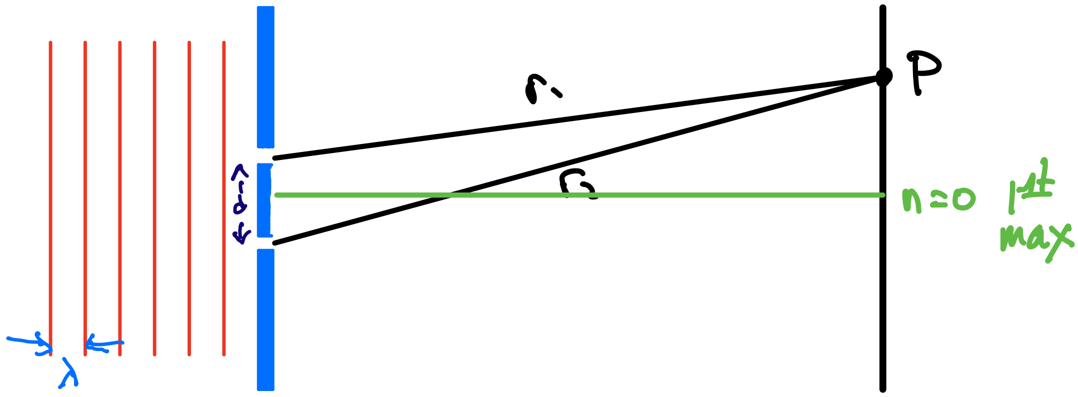
$\boxed{\Delta r = \left(n+\frac{1}{2}\right)\lambda}$

put screen on right side of slits, dist R away



Interference pattern at point P on screen

will be due to path difference from each slit (each wavelet) to that point



at point P will get:

constructive: $\Delta r = n\lambda$

destructive: $\Delta r = (n+\frac{1}{2})\lambda$

⇒ so there should be series of points up and down screen where interference goes from min to max to min etc.

MAX: when $\Delta r = n\lambda$

MIN: when $\Delta r = (n+\frac{1}{2})\lambda$

n=0 is in middle where $\Delta r = 0$ (same path)

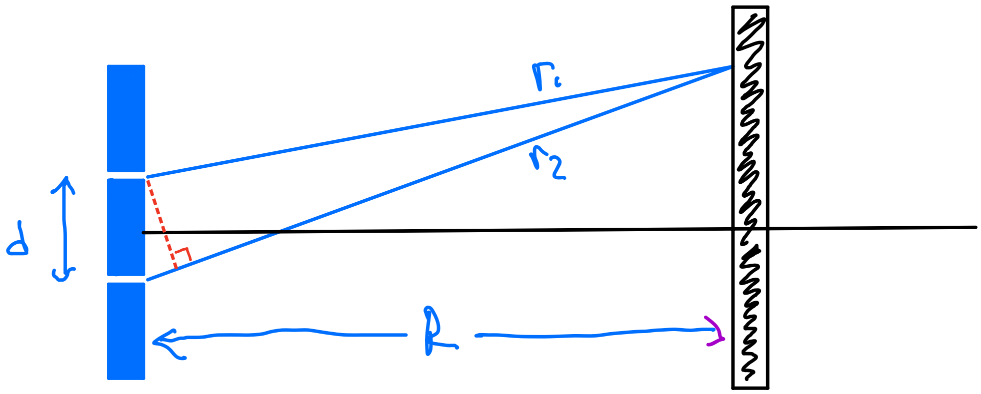
⇒ 1st interference max

Next: calculate angles for MIN & MAX

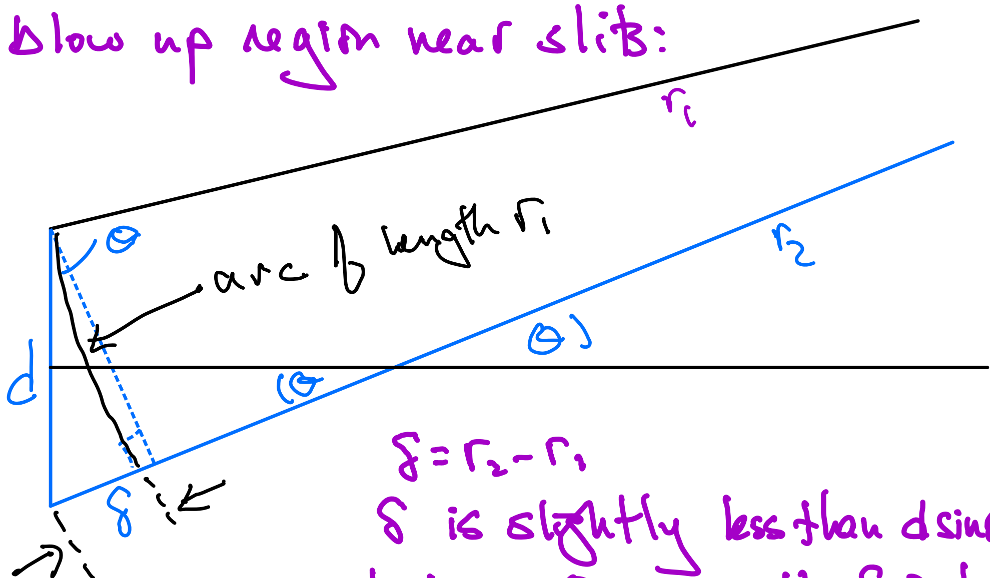
At the screen:

MAX intensity when $\Delta r = n\lambda$

MIN " " $\Delta r = (n + \frac{1}{2})\lambda$



Blow up region near slits:



$\delta = r_2 - r_1$

δ is slightly less than $d\sin\theta$

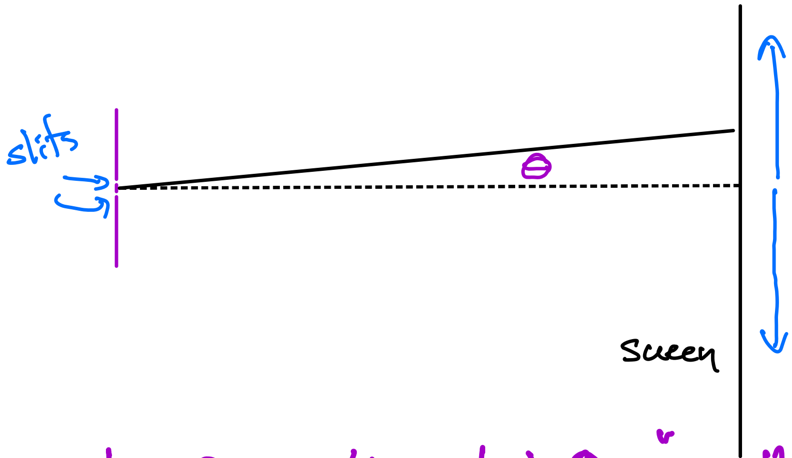
but as $\theta \to$ small, $\delta \to d\sin\theta$

$\Rightarrow$ and as $R >> d$, then $\theta \to$ small

$\Rightarrow$ we will always use this small angle limit

$\Delta r = \delta = d\sin\theta = n\lambda$ condition for constructive interference

similarly $\delta = d\sin\theta = (n + \frac{1}{2})\lambda$ condition for destructive screen at $R \gg d$, calculate coordinate "y" on screen



y measures position on screen

$\tan\theta = y/R$ but $\theta \sim$ "small" so $\tan\theta \sim \sin\theta$

$\sin\theta = y/R \Rightarrow y = R\sin\theta$

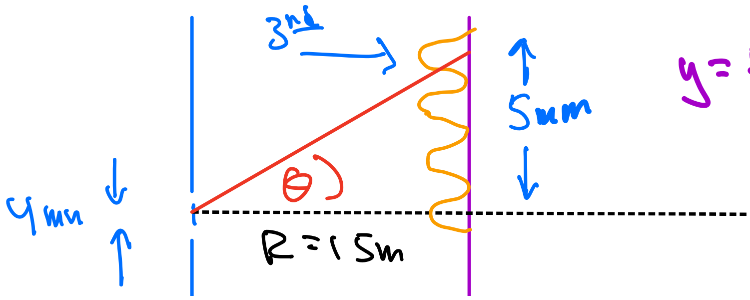
and $d\sin\theta = n\lambda$

so $y = \dfrac{Rn\lambda}{d}$ $n = 0, \pm1, \pm2, \ldots$

this tells you where the maximum intensities from interference

Example: I have a light source with unknown λ, how can I find λ?

- place a screen 15 m beyond slits separated by 4 mm
- you measure the 3rd peak above central peak at 5 mm



$y = \dfrac{Rn\lambda}{d} \Rightarrow \lambda = \dfrac{yd}{nR}$

$= \dfrac{5\times10^{-3} \cdot 4\times10^{-3}}{3 \cdot 15}$

$= 4.4\times10^{-7}$ m

$= 0.44 \times 10^{-6}$ m

$= 440$ nm

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ex: what is width of each peak on screen?

1st min is at $d\sin\theta_1 = \frac{1}{2}\lambda$ $(n=0)$

2nd " " " $d\sin\theta_2 = \frac{3}{2}\lambda$ $(n=1)$

400 3/5

and $\sin\theta_1 = y_1/R$

$\sin\theta_2 = y_2/R$

so $\frac{d y_1}{R} = \frac{\lambda}{2} \Rightarrow y_1 = \frac{\lambda R}{2d}$

$\frac{d y_2}{R} = \frac{3\lambda}{2} \Rightarrow y_2 = \frac{3\lambda R}{2d} = 3y_1$

$\Delta y = y_2 - y_1 = 3y_1 - y_1 = 2y_1 = \frac{\lambda R}{d}$

$= \frac{440 \times 10^{-9} \times 15}{4 \times 10^{-3}} = 1.66 \times 10^{-3}$ m

≈ 1.65 mm

shortcut: $y = n\frac{R\lambda}{d}$

$\Rightarrow \Delta y = \Delta n \cdot \frac{R\lambda}{d}$

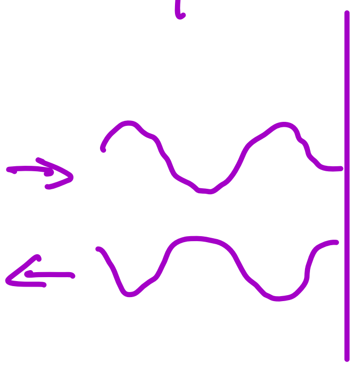
here we want $\Delta n = 1$ to get Δy for 1 maxima

so $\Delta y = \frac{R\lambda}{d} = \frac{440 \times 10^{-9} \times 1.5}{4 \times 10^{-3}} = 165$ mm

Thin film interference

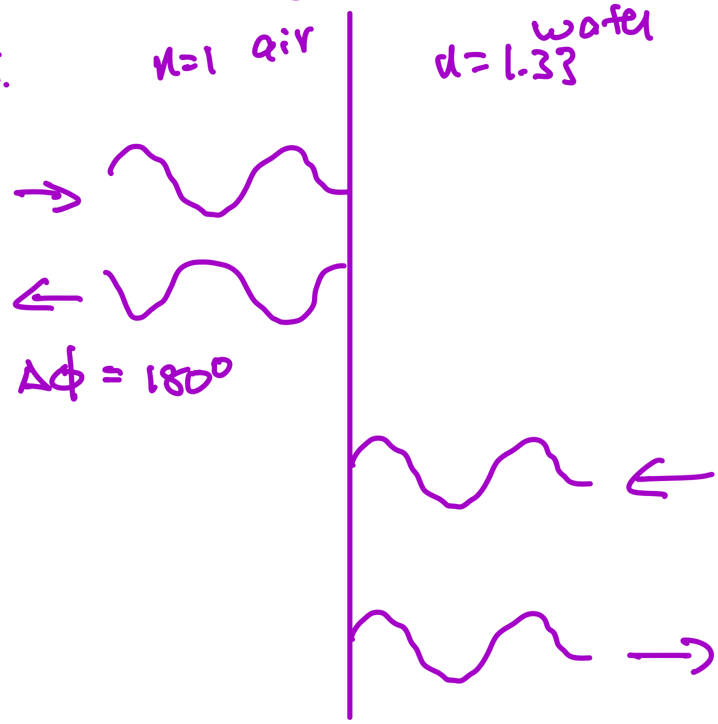
Light waves reflected at mirror:

reflected wave phase changes by 180° (π)



At boundary where there is refraction + reflection, the reflected wave has phase inversion if n is increasing (like a mirror)

ex. $n=1$ air | $n=1.33$ water



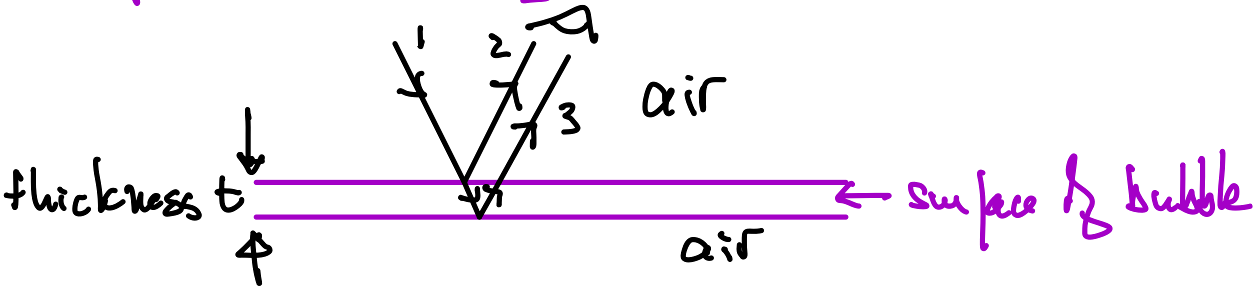
180° flip

$\Delta\phi = 180^\circ$

no phase change

$\Delta\phi = 0$

Soap bubble : soapy water with air on both sides



wave 2: $\Delta\phi_2 = 180° = \pi$ relative to wave 1

wave 1 refracts ($\Delta\phi = 0$) and reflects off bottom surface where n <u>decreases</u>

$\Delta\phi = 0$,

also there's a phase diff due to the path Δr given by $\Delta\phi_1 = k\Delta r$

where $\Delta r = 2t$ (ignore angle)

total phase difference

$$\Delta\phi_{tot} = \Delta\phi_1 - \Delta\phi_2 = k \cdot 2t - \pi$$

trick: can add 2π to $\Delta\phi_{tot}$ w/no change

$$\Delta\phi_{tot} = k \cdot 2t + \pi$$

if $2t < \lambda$ then path difference can be neglected

so $\Delta\phi$ between 2 & 3 is π

$\Rightarrow$ waves interfere destructively

$\Rightarrow$ this is why you see dark layers if $t \sim \lambda$ or bigger, can't ignore path diff

ex: red light $\lambda = 650$ nm, $n = 1.33$ in bubble

calculate t so that red light interferes
constructively (you see it)

$\Delta\phi$ from thickness path diff

$$\Delta\phi = k\Delta r = k \cdot 2t$$

where $k = \frac{2\pi}{\lambda_n}$ λ_n is wave length inside bubble where the extra path is

you get $180° = \pi$ from 1st reflection

so want another π phase shift due to Δr

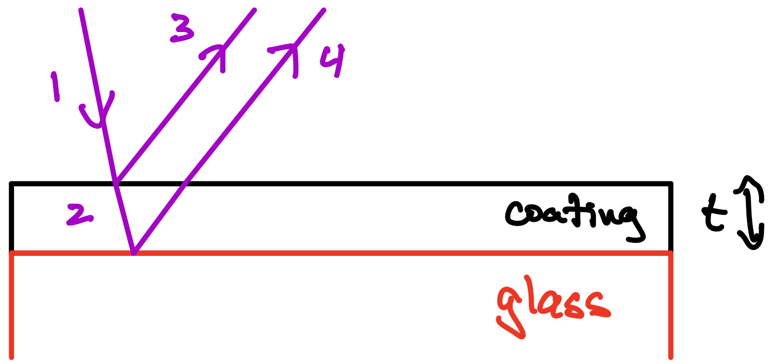
$$\frac{2\pi}{\lambda_n} \cdot 2t = \pi \Rightarrow$$

$$t = \frac{\lambda_n}{4} = \frac{\lambda/n}{4} = \frac{650/1.33}{4} = 122 \text{ nm}$$

This is why you see colors on soap bubbles - non uniform thickness of bubble

$\Rightarrow$ Can use this to coat lenses so they don't reflect certain colors ex: lens $n = 1.52$ and want to coat so $\lambda = 500\,nm$ is not reflected.

coating $n_c = 1.38$



wave 1: n increases so $\phi = \pi$ for wave 3

wave 2: path diff 2t and $\phi = \pi$ at coating-glass surface

total $\Delta\phi = \pi$ 1st reflection

$+ \pi$ 2nd reflection

$+ k_n \cdot 2t$ path diff

so $2\pi + \frac{2\pi}{\lambda_n} \cdot 2t \Rightarrow 2\pi$ gives in phase so can ignore it

so want $\frac{2\pi}{\lambda_n} \cdot 2t = \pi$ to get destructive interference

$t = \frac{\lambda_n}{4} = \frac{500/1.38}{4} = 90.6\,nm$

200 3/7 400 3/10